Theory of Radiometer Forces

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Abstract

This document is an essay in the translation of "Zur Theorie der Radiometerkäfte" by Albert Einstein. The original document was found in: in German. Rather than adding notes with corrections, the corrections have been made and original versions put into notes. This makes a less confusing argument in the opinion of the author The translation is direct, except where notes and corrections have been incorporated into the text.

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On the basis of schematic assumptions about the mechanism of molecular evolution in gases, the forces are approximated which act on the mean free path λ of small bodies and on the boundary zone against λ large bodies in a warm flow. The theory of force effects and pressure differences in gases caused by temperature differences has been clarified by Knudsen in the case that the free path is large compared to the decisive vessel dimensions. On the other hand, there is still a great deal of uncertainty about the causes of the thermo-forces in traps, where the free path is of the same order of magnitude or smaller than the decisive framing dimesions. In the following I will give a more qualitative consideration of the circumstances prevailing here, taking into account the quantitative only of the order of magnitude. If the considerations given here are also quite elementary in nature, then they have helped me out with uncertainties, and I hope that some readers will be detained with this brief exposition.

1. On the path of small bodies in a heat gradient

At first we think of an extended gas, in which the positive x-axis is progressively formed by a stationary homogeneous heat-current. Molecular motion is largely schematized, ascribing the same velocity to all particles, u, except for the small differences that we consider to be a schematic account of the thermal flow. Against the path of small bodies in a gradient of heat. At first we think of an extended gas, in which the positive x-axis is progressively formed by a stationary homogeneous heat-current. Furthermore, if we calculate as that the molecules move only slowly along the coordinate axes. We can treat the path length λ as a constant length. All these approximations only bring us insignificant errors in the numerical coefficients in the formulas, without disturbing the conception of the essential relationships. We first consider the molecular motion through a surface element of σ perpendicular to the x-axis, against λ. Mass flow should be zero, therefore, exactly the same number of molecules pass through σ per second in both directions, namely:

$$\frac{1}{6}n\sigma u \tag{1}$$

Molecules, where n denotes the number of molecules per unit volume. In order to do justice to the fact of the warm flow, we must assume that the velocity of the molecules u_+ in the direction of the positive x-axis is slightly larger than u; the correspondingly defined u_- must be correspondingly slightly smaller than u. The heat flow σ f through the surface element is given by:

$$\sigma f = \frac{1}{6}n\sigma u\left(\frac{m}{2}u+^2 - \frac{m}{2}u-^2\right) \tag{2}$$

Considering Equation 2 in the context of:

$$\frac{1}{2}mu^2 = \frac{3}{2}xT$$

as well as the fact that for the molecular velocity u_+ or u_- temperatures are relevant at the places where the last collision took place ($\lambda =$ mean free path), then instead of 2:

$$f = -\frac{n}{2}x\lambda u\frac{\delta T}{\delta x} \tag{2a}$$

Now consider instead the surface element a small body of the surface extension σ. The molecules colliding with it in the x-direction give an impulse overflow, K in the direction of the positive x-direction:

$$K = \frac{1}{6}n\sigma u(mu_+ - mu_-) \tag{3}$$

If one neglects the fact that on exiting the body by the colliding molecules another impulse effect on the body arises, which constitutes a certain fraction of the just calculated, K is also the moving force acting on the body. It follows from 2 and 3 that u_+ and u_- deviate only slightly from u:

$$K = \frac{\sigma f}{u} = -\frac{1}{2}p\frac{\lambda}{T}\frac{\partial T}{\partial x}\sigma \tag{3a}$$

Where p is the gas pressure. In this formula, as in 2, f refers only to the part of the warm flow that is based on the translational motion of the molecules.

This force, K, will move a particle in the direction of the positive x-axis when it is free. In order to know the velocity, v, of this motion, we need only calculate the frictional force $K^{'}$, which is exerted by the gas on the particle when it is moved by the gas at the velocity v. This frictional force arises essentially from the fact that the body communicates on average to each colliding molecule the impuls mv. By execution of the corresponding elementary bill one receives:

$$K^{'} = -\frac{4}{3}n\sigma u.mv \tag{4}$$

The equation of K and $-K^{'}$ yields:

$$v = \frac{1}{4}\frac{f}{RTn} = -\frac{1}{8}u\frac{\lambda}{T}\frac{\partial T}{\partial x} = \frac{1}{4}\frac{f}{p} \tag{5}$$

These velocities, which, as long as the particles are small in size, are independent of the particle size, can become quite considerable.

If $\lambda = 0.1cm$ and $\frac{\partial T}{\partial x} = 30$, $T = 300$ and H_2 gas then the results obtained about 1m per second, at atmospheric pressure and otherwise equal ratios over 0.1mm per second.

These forces play a crucial role, for example in the deposition of the frost and in electric heaters used to purify the air of smoke particles.

2. In the case of a small hole in a thin wall transverse to the heat flow

We come now to a phenomenon which is the counterpart to the one we have just considered. The reasoning of Section 1 was based mainly on the fact that inside a flow-free gas the number of molecules meeting a surface element from both sides is equal. We come now to a phenomenon which is the counterpart to the one we have just considered. The reasoning of §1 was based mainly on the fact that inside a flow-free gas the number of molecules meeting a surface element from both sides is equal. We put it this way: the condition of flow equality is fulfilled in the interior of the heat-permeated gas. The calculated force exerted on a particle resulted from the fact that the same number of molecules on the front and back of the particle carry different momentum.

This “flow equality” in the gas interior now faces a “pressure equator” with respect to the walls of the gas space. Namely, it is well known and easy to show that everywhere on the walls of the gas space even with uneven temperature distribution in the gases, equal compressive forces per unit area must act, if only the considered wall parts are sufficiently large compared to the free path. Which is taken by itself to be sufficiently uniformly tempered and by Gasqerschnitte are separated from each other, which are large in all dimensions against the free path. Then the concepts and laws of hydrostatics of the continuums are applicable.

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In the gas considered above, there is a flat plate oriented perpendicular to the heat flow, *i.e.* parallel to the y-z plane. Let it be large compared to the free path length, and its edges should have reasonable distance from the remaining walls of the gas space, which are large compared to λ. Also, there is pressure equality despite the presence of the heat flow.

Any molecule that hits the plate from the side of the negative x may have the velocity u_n and leave the plate in the negative x-direction at the velocity u. v_n of such impacts may occur per unit area and time. u_p, u and v_p are the corresponding values for the other side of the plate. It is assumed here that the velocities of the molecules leaving on both sides of the plate after the collision are the same. The condition of pressure equality is then:

$$\frac{p}{m} = v_n(u + u_n) = v_p(u + u_p) \tag{6}$$

Further, the heat flow on both sides of the plate must be the same, which is expressed by the equation.

$$\frac{2f}{m} = v_n(u_n^2 - u^2) = v_p(u^2 - u_p^2) \tag{7}$$

By dividing these two equations one obtains first:

$$\frac{2f}{p} = u_n - u = u - u_p \tag{8}$$

Substituting this in Eqn. 6, one gets by replacing $\frac{v_n+v_p}{2}$ by v and replacing this with $\frac{nu}{6}$:

$$v_p - v_n = \frac{1}{3}\frac{nf}{p} \tag{9}$$

If there is an opening in the plate that is small in relation to the free path σ, then $(v_p - v_n)\sigma$ molecules must pass in the direction of the decreasing x per unit time more than in the opposite direction $v_p - v_n$, *i.e.* the intensity of an opening (recurrent) molecular flow, the apparent flow velocity of which is given by the equation

$$v_p - v_n = -nv \tag{10}$$

from 9 and 10 follows:

$$v = -\frac{1}{3}\frac{f}{p} \tag{10a}$$

Eqn. (10a) forms the counterpart to equation (5)

3. Cause of certain radiometer forces in denser gases

These results are essentially still in the area of the laws of Knudsen gas, in which the effective body dimensions are small compared to the free path in the gas. However, they also provide the key to certain radiometer detections in denser gases.

A sheet whose measurements are large compared to the free path length λ, lies in the interior of the gas, perpendicular to a heat flow. Similarly, the distances of the vessel walls from the edges of the sheet are large compared to λ. Sufficiently far from the edge of the sheet, the pressure balance will then prevail on the sheet, but far enough outside the leaflet the conditions studied in §1 will prevail which will cause a small body to experience the compressive force $\frac{\sigma f}{u}$ against λ.

At the edge of the sheet a gradual transition will take place between these two states of the gas whose width is of the order of magnitude λ. It is thus on the unit length of the plate edge a force of the order of magnitude:

$$K = \frac{f\lambda}{u} = -\frac{1}{2}p\frac{\lambda^2}{T}\frac{\partial T}{\partial x} \tag{11}$$

as long as the dimensions of the leaflet are large against the free path.

The case of the sheet warmed on one side is similar in that here too a marginal zone of the indicated width λ, will be present in which the pressure equality on both leaflet sides is not fulfilled. In this case, which is less suitable for a quantitative examination, I find the expression for the force acting on the unit of length of the edge

$$K = -p\lambda\frac{\Delta T}{T} \tag{11a}$$

Which, of course, is probably only valid for the order of magnitude.

Another cause of radiometer forces is the rate of sliding a wall gives to gases at tangential temperature gradients. This phenomenon, which has already been opened up by Maxwell in a theatrical way and was found by Knudsen independently, is currently being worked on by Messrs. Hettener and Czerni.

3. (Alternative Ending) - Wall with Temperature Jump

In the event that a termpeature difference of the two wall surfaces is maintained by irradiation or otherwise, we obtain analogous effects, even if no heat flow in the gas is taken into account. Again, we reiterated the case that the communication of the guest parts on either side of the wall is such that pressure equalization occurs almost completely. This corresponds to the equation

$$\frac{p}{m} = 2v_n u_n = 2v_p u_p \tag{12}$$

becomes:

$$\Delta v = v_p - v_n \qquad v = \frac{1}{2}(v_p + v_n)$$

so as to get:

$$\frac{\Delta v}{v} = -\frac{\Delta T}{2T} \tag{11a}$$

$$-\Delta v = nv$$

$$v = \frac{1}{3}nu$$

If there is a small opening σ in the wall, then Δv denotes the intensity of the molecular flow through the opening. With regard to the relation:

$$-\Delta v = nv$$

$$v = \frac{1}{6}nu$$

From this you get:

$$v = \frac{1}{12}u\frac{\Delta T}{T} \tag{11b}$$

which relation in this case takes the place of (10a). To make the comparison easier to understand, we note that (10a) can be also written in the From:

$$v = -\frac{1}{12}u\frac{\lambda\frac{\partial T}{\partial x}}{T} \tag{9b}$$

4. Principles of the previous sections

If a gas is in thermodynamic equilibrium, the three conditions are fulfilled at each location in each direction.

a) $v_+ = v_-$ (flows balance)

b) $mu_+v_+ = mu_-v_-$(forces balance)

c) $\frac{1}{2}mu_+^2v_+ = \frac{1}{2}mu_-^2v_-$ (heat flow balances)

If a heat flow takes place, then c) is must be replaced by:

c') $f = \frac{1}{2}m(u_+^2v_+) - u_-^2v_-$

If $f \neq 0$, *i.e* if there is a heat flow, then a) and b) can not be fulfilled at the same time. There is no flow in the free gases, so a) is fulfilled. In place of b) we must use:

b') $i = mu_+ v_+ - mu_- v_- = \frac{f}{u}$

So connected to the heat flow, a pulse current i, which manifests itself as a force λ against relatively small particles, that enter this the temperature gradient.

If we consider that there is a large sheet λ, which is placed with its surface perpendicular to the heat flow, so takes place at pressure equilibrium. It is possible approximately, though not exactly, that b) is satisfied. But a) is not fulfilled, since a), b), c ') are incompatible with each other. One finds rather

a') $v_+ - v_- = -\frac{2f}{3kT} = -\frac{2}{3}\frac{nf}{3p}$

which matches except for the number factor with (9).